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EVALUATION OF NUMERICAL MODELS FOR A FLOATING

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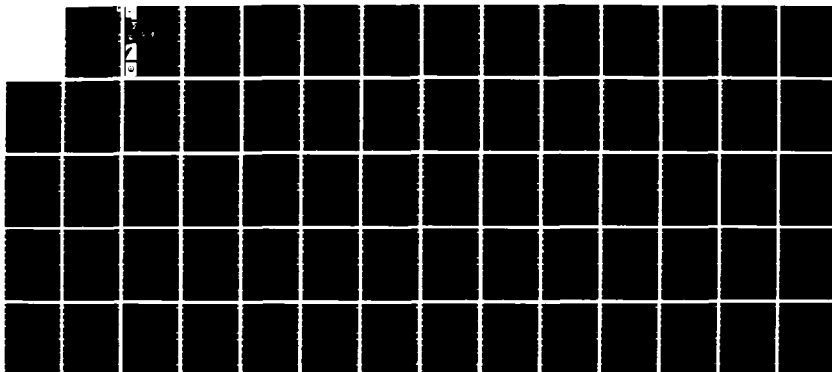
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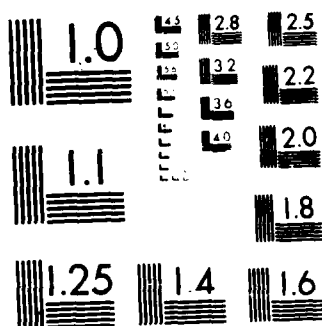
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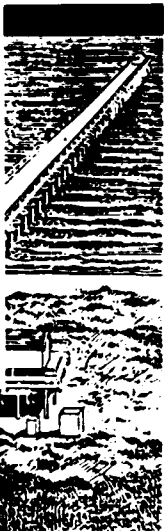


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EVALUATION OF NUMERICAL MODELS FOR A FLOATING BREAKWATER

by

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19 ABSTRACT (Continue on reverse if necessary and identify by block number) A large number of existing state-of-the-art numerical models were reviewed for the purpose of selecting an efficient model to be used for routine analysis of wave attenuation and mooring forces associated with a floating breakwater. The models reviewed came from five basic categories: Eigen Function Expansion Models, Green Function Models, Hybrid Green Function Models, Finite Element Models and other models. The critical parameters in the selection process were (a) computational efficiency; (b) ability to accommodate three-dimensional spectra as the input wave; and (c) versatility relative to breakwater geometry and water depths. Based on these criteria, a Hybrid Green Function Model was selected as the optimum numerical model, followed by a Green Function Model. Recommendations are presented which include the acquisition and adaptation of the selected model and the comparison of output with corresponding prototype data. The two-dimensional model also was recommended for use (Continued)			
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19. ABSTRACT (Continued).

as a fast diagnostic assessment tool. During test runs, investigators should pay special attention to the development of optimum procedures for determining the mooring force. Results of prototype tests performed by the Coastal Engineering Research Center and the Japanese Ministry of Transport can be used to support test runs. .

Preface

This report documents results of an evaluation of 24 state-of-the-art numerical models suitable for floating breakwater applications. The work was sponsored through funds provided to the Coastal Engineering Research Center (CERC), US Army Engineer Waterways Experiment Station (WES) by the Civil Works Directorate, Office, Chief of Engineers (OCE), US Army Corps of Engineers, under the Coastal Structures Evaluation and Design Program Work Unit 31679 titled "Design of Floating Breakwaters." OCE point of contact was Mr. Jesse Pfeiffer, Jr.

The report was prepared under WES Purchase Order No. DACW39-85-M-4381. Mr. Kozo Bando, an employee of Kajima Corporation, Japan, performed this study as the senior author while serving as a resident engineer at TEKMARINE, Inc. The principal investigator of the evaluation was Dr. Choule J. Sonu, TEKMARINE, Inc. Mr. Toshihiko Takahashi of Kajima Corporation, Japan, kindly provided useful information to support the study. Dr. H. S. Chen, Research Division, CERC, offered helpful technical comments during discussions of evaluation results.

This effort was coordinated by Mr. Peter J. Grace, Research Hydraulic Engineer, Wave Research Branch, Wave Dynamics Division (WDD), under general supervision of Dr. James R. Houston and Mr. Charles C. Calhoun, Jr., Chief and Assistant Chief, CERC, respectively, and under direct supervision of Messrs. C. E. Chatham, Chief, WDD, and D. D. Davidson, Chief, Wave Research Branch.

During preparation of this report, COL Allen F. Grum, USA, was Director of WES. During publication of this report, COL Dwayne G. Lee, CE, was Commander and Director. Dr. Robert W. Whalin was Technical Director.



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**EVALUATION OF
NUMERICAL MODELS
FOR A FLOATING BREAKWATER**

1. Scope of Study

The objective of this study was to identify an efficient numerical model which will analyze and compute wave attenuation and mooring forces related to a floating breakwater. Of particular interest was the determination of wave height attenuation in the lee of a floating breakwater in a random sea, with sufficient accuracy and computational efficiency. Other key attributes desired in the numerical model included, but were not limited to:

- a) flexibility in handling arbitrary structure shapes and water depths;
- b) availability of the code for unrestricted use by the Corps of Engineers; and
- c) a track record of acceptable performance.

In the case that a model meeting all these requirements could not be found, the study would recommend models which could be modified and/or improved upon to meet the Corps needs.

2. Existing Models

An initial review of the current literature disclosed at least 24 separate models dealing with the hydrodynamic processes involving a floating body in the ocean environment. These are listed in Table 1. Although further searches will undoubtedly expand the list, it became apparent that the 24 models could be grouped into the five principal categories listed below, depending upon the approaches used in evaluating the velocity potential:

TABLE 1
LIST OF NUMERICAL MODELS FOR ANALYSIS OF FLOATING BREAKWATER

NUMERICAL APPROACHES	AUTHORS	FLOATING STRUCTURE			APPLICABLE WATER DEPTH
		SHAPE	CROSS-SECTION	DIMENSION	
Eigen Function Expansion Method (EFEM)	1 Mei and Black (1969)	—	□	2	Constant
	2 Garrett (1971)		○	3	"
	3 Black <u>et al.</u> (1971)	—	○	2,3	"
	*4 Ijima <u>et al.</u> (1972a)	—	□	2	"
	5 Ijima <u>et al.</u> (1972b)		○	3	"
	*6 Ito and Chiba (1972)	—	□	2	"
	7 Yeung (1981)		○	3	"
Green Function Method (GFM)	8 Kim (1965)		A	3	Constant
	*9 Garrison (1974, '78)		A	3	"
	*10 Adee and Martin (1974)	—	A	2	Infinite
	*11 Georgiadis and Hartz (1982)	—	A	3	Variable
Hybrid Green Function Method (HGFM)	*12 Ijima <u>et al.</u> (1975)		□ ○	3	Constant
	*13 Ijima <u>et al.</u> (1976)	—	A	2	Variable
	14 Yamamoto <u>et al.</u> (1980)	—	A	2	"
	*15 Kiyokawa <u>et al.</u> (1983)	—	A	2	"
Finite Element Method (FEM)	16 Zienkiewicz <u>et al.</u> (1969, '77)		A	3	Variable
	17 Chen <u>et al.</u> (1974)		A	2	"
	*18 Yue <u>et al.</u> (1978)		A	3	"
	19 Hara (1978)		A	3	"
	*20 Bando <u>et al.</u> (1984 a, b)		A	3	"
OTHERS	21 Ursell (1949)	—	○	2	Infinite
	22 Yu and Ursell (1961)	—	○	2	Constant
	23 Wang (1966)	Sphere		3	"
	24 Vugts (1966)	—	Various	2	Infinite

Note: — = Horizontal ○ = Circular
 | = Vertical □ = Rectangular
 A = Arbitrary ◊ = Elliptical

* Models retained for additional detailed evaluation in this study.

- o Eigen Function Expansion Method (EFEM)
- o Green Function Method (GFM)
- o Hybrid Green Function Method (HGFM)
- o Finite Element Method (FEM)
- o Others

The Eigen Function Expansion Method (EFEM) determines the velocity potential through analytical expressions in the form of Eigen Function expansion. Applicable only to structure shapes of simple geometry and constant water depths, the EFEM is the simplest numerical model of the five categories mentioned. It is also basically a two-dimensional model.

The Green Function Method (GFM) arrives at the velocity potential by solving an integral equation which contains the Green Function. The integral equation is defined on the structure surface from which the terms "boundary integral" or "boundary element" methods derive. Since the numerical evaluation of the governing integral equation is performed on the discretized elements of the structure surface, the computational load in the Green Function Method could become quite burdensome depending upon the complexity of the structure shape.

The Hybrid Green Function Method (HGFM) combines some features of both the Green Function Method and the Eigen Function Expansion Method. The purpose is to reduce the computational burden in cases of complex breakwater geometry and variable water depths. In this method, the domain of interest is separated into interior and exterior sub-regions to simplify the boundary conditions, and the velocity potential is expressed in each sub-region using both the Eigen Function series and a Green Function.

The Finite Element Method (FEM) determines the velocity potential by evaluating the governing integral equation in the entire domain of the fluid. In the Finite Element Method, therefore, the entire domain is discretized into small elements, while in the Green Function Method only the structure boundary is discretized. Many existing FEM models employ some degree of hybrid technique in order to relax the radiation condition imposed at infinity and reduce the computational load. This modification, sometimes called the "Hybrid Element" method, could confine the domain to be discretized in a limited region by employing analytical representations in the far field.

Other Models typically employ a series of singular functions to represent the velocity potentials associated with body motions. This method was the forerunner among various approaches which were used to obtain the added mass and damping coefficients in ship motion problems.

3. Mathematical Procedures

3.1 Two Assumptions

In describing wave motion in the presence of a floating breakwater, there are certain mathematical features common to all the numerical models. One is the use of velocity potential; another is the assumption of linearity for fluid motion.

The use of velocity potential is predicated on the assumption that the fluid motion is irrotational. The assumption guarantees the existence of a scalar function called "velocity potential":

$$\phi(x, y, z) \quad (1)$$

which satisfies the following condition:

$$u = \frac{\partial \phi}{\partial x} , \quad v = \frac{\partial \phi}{\partial y} , \quad w = \frac{\partial \phi}{\partial z} . \quad (2)$$

Thus, once the velocity potential ϕ is determined, all three unknowns u , v and w can be determined readily through Equation (2). This is convenient: instead of pursuing three unknowns simultaneously, one may seek to determine a single unknown, ϕ . In order to determine the velocity potential, one must solve a continuity equation, which, in the irrotational fluid motion, takes the form of Laplacian:

$$\nabla^2 \phi = 0 \quad (3)$$

under proper boundary conditions. It is mainly in the procedures for solving Eq. (3) that the major distinctions between different numerical models arise. The remaining unknown, pressure (p), can now be determined by substituting ϕ into the general Bernoulli equation (i.e., pressure equation) of the form:

$$\text{grad} \left(\frac{\partial \phi}{\partial t} + \frac{1}{2} q^2 + \int \frac{dp}{\rho} + \Pi \right) = 0 \quad (4)$$

in which

$$q^2 = u^2 + v^2 + w^2 \cong 0 \text{ (linearity assumption)}$$

Π = potential of the system

p = pressure

ρ = fluid density

$$\text{grad} = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

With velocity potentials and pressure thus determined, the equations of motion are completely described, which can now be solved for body response and then for both wave height distribution and forces on the structure. Although there are some variations, most of the existing numerical models follow the computational steps outlined above.

Another feature which is common to the various numerical models is the assumption that the fluid system is linear. Physically, this assumption means that the amplitudes of the wave and structural motions are small. Mathematically, this assumption means that the velocity potentials are additive. For instance, if ϕ_1 denotes the velocity potential associated with a unit heave of the structure, the velocity potential associated with an arbitrary heave x_1 will be $x_1 \phi_1$. If we let ϕ_2 denote the velocity potential associated with a unit sway of the structure, the velocity potential arising from simultaneous heave x_1 and sway x_2 will be $x_1 \phi_1 + x_2 \phi_2$, and so on.

Using the additive property of the velocity potential in a linear system, one may first derive velocity potentials for all unit motions and then combine them, linearly, as needed to match the real scenarios. There are usually eight such velocity potentials: ϕ_0 for incident wave, ϕ_1 through ϕ_6 , respectively, for

the heave, sway, surge, roll, pitch and yaw motions of the structure in calm water, and ϕ_7 for scattered wave. While the linearity assumption basically implies that the motions involved are small, the solutions are usually able to describe moderate-sized motions adequately as well.

3.2 Eigen Function Expansion Method (EFEM)

The EFEM can be considered the simplest numerical model among the various models discussed in this report, and thus is applied only to simple shaped structures in constant water depth. The flow diagram for the EFEM is shown in Figure 1. This method seeks to determine the velocity potential in closed form by considering simplified boundary conditions. For this purpose, the domain of interest is divided into two sub-regions of simple geometry as shown in Figure 2: the "interior" sub-region beneath the floating body bounded by the body, the sea bottom and the artificial borders, and the "exterior" sub-region with the free surface extending to infinity.

In both of the sub-regions, the velocity potential can be represented by the form of eigen function expansion with analytical expressions which satisfy respective boundary conditions except at the artificial boundary. These expansions include an infinite number of unknown coefficients which are determined by requiring continuity of pressure and velocity across the artificial boundary using either the variational method (Mei and Black, 1969; Black et al., 1971) or the Galerkin method (Garrett, 1971; Ijima et al., 1972a, b).

The EFEM was applied to a fixed floating body by Mei and Black (1969) and Garrett (1971) and to an oscillating floating body in a calm sea by Black et al., (1971) and Yeung (1981). For the case of

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EXTERIOR SUBREGION

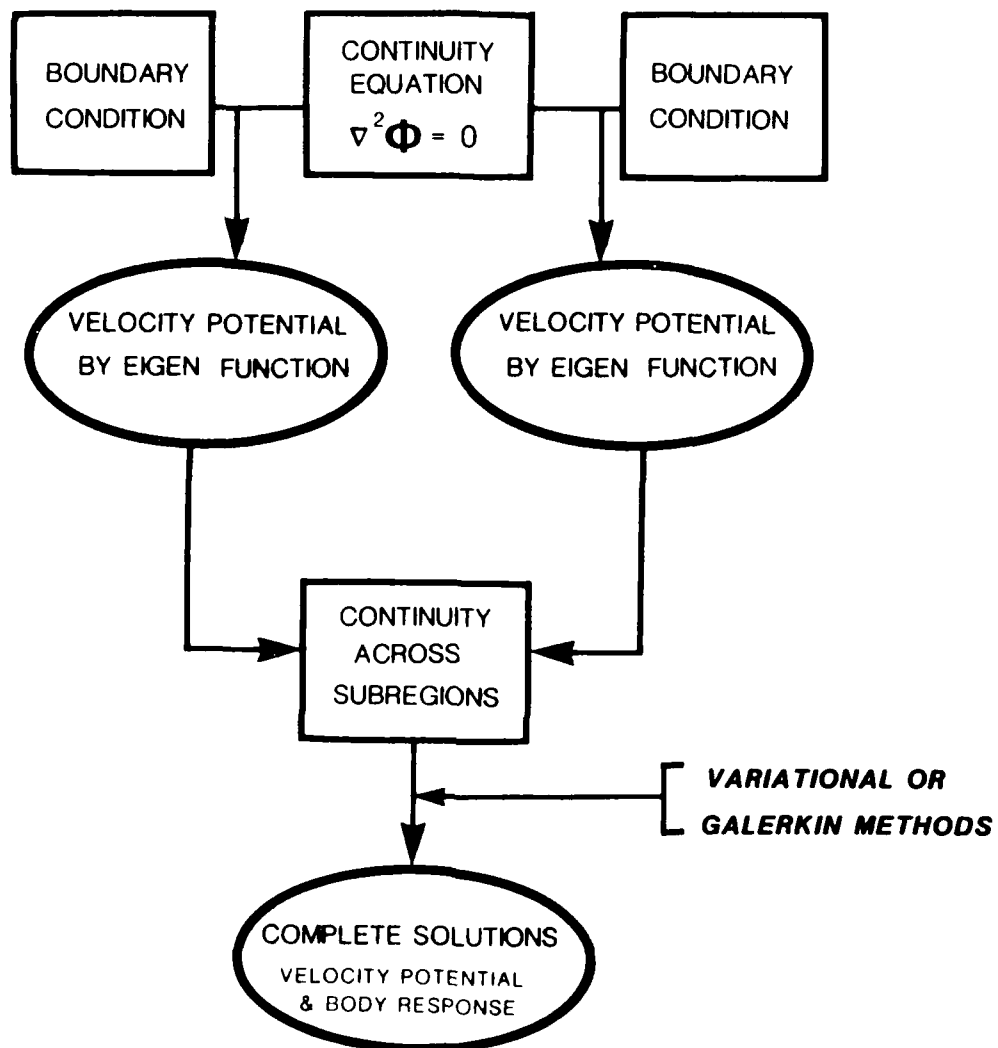
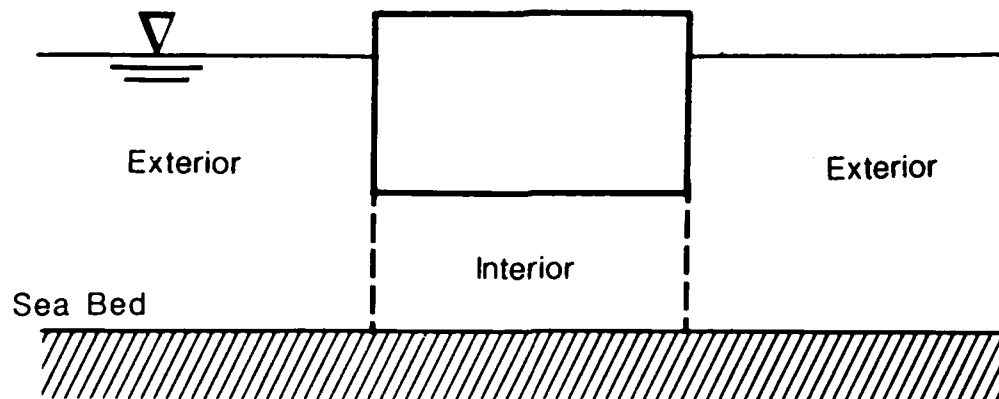


Figure 1. Flow Diagram for Eigen Function Expansion Method (EFEM)

Elevation View



Plan View

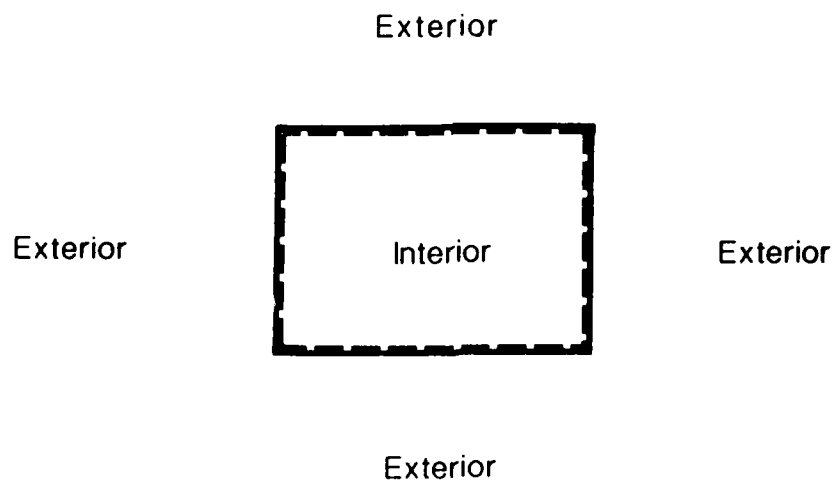


Figure 2. Partition of Domain into Subregions of Simple Geometry.

a floating body in the presence of waves, Ijima et al., (1972a, b) developed a model in which velocity potential and body response are determined simultaneously.

The model developed by Ito and Chiba (1972) essentially represents an approximate method employing only the first term of an infinite series in the Ijima model. Their method demonstrated fairly consistent agreement with empirical experiments.

3.3 Green Function Method (GFM)

The Green Function, expressed $G(x, \xi)$, defines a velocity potential at a position x when a unit charge is placed at a source point ξ . If source strength is $f(\xi)$ instead of unity, velocity potential at x should be $f(\xi) \cdot G(x, \xi)$. If charges of strength $f(\xi)$ are distributed over a boundary S , velocity potential $\phi(x)$ at x is expressed by

$$\phi(x) = \int_S f(\xi) \cdot G(x, \xi) ds \quad (5)$$

Various forms of Green function are known. One may be selected that allows velocity potential in accordance with the above definition to satisfy the continuity equation, independent of $f(\xi)$.

$$\nabla^2 \phi = 0 \quad (6)$$

The selected Green Function may or may not satisfy the following series of boundary conditions, such as boundary condition at sea surface:

$$\frac{\partial \phi}{\partial z} = (\omega^2 / g) \phi \quad (7)$$

boundary condition at sea bed:

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{at } z = -h \quad (8)$$

and the radiation condition at $r \rightarrow \infty$. The velocity potential thus defined contains ambiguity because $f(\xi)$ is not known. This function is determined numerically by satisfying the remaining boundary conditions and by substituting the kinematic boundary condition,

$$\frac{\partial \phi}{\partial n} = V_n \quad (9)$$

into the integral equation (5). These two-step procedures completely define the velocity potential ϕ .

Kim (1965) and Garrison (1974, 1978) presented fully three-dimensional Green function models applicable to floating breakwaters of arbitrary geometry in constant water depth. Georgiadis (1982) studied the response of an infinite floating bridge subjected to oblique incident waves as a two-dimensional case using the cosine projection for obliquely incident waves. This approach is unable to handle wave diffraction around the edge of the breakwater.

The computational procedures used in the GFM are illustrated in the flow diagrams shown in Figures 3, 4, and 5. In Figure 3, the entire computation is categorized into two successive sequences consisting of "unit response analysis" and "body response analysis".

The unit response analysis determines the velocity potentials associated with unit motions of the floating body in calm water and unit incident wave. The end products of this sequence of computations are velocity potentials for the unit of heave, sway, surge, roll, pitch, yaw (ϕ_1 through ϕ_6), velocity potential for scattered wave (ϕ_7), plus added mass, damping coefficient, and exciting force. Detailed steps in the unit response analysis are shown in Figure 4.

The body response analysis essentially involves solving equations of motion using the output from the preceding unit response analysis. As shown in detail in Figure 5, it first establishes the finite motions of body response x_1 through x_6 (corresponding to heave, sway, surge, roll, pitch and yaw) under given finite wave amplitude H . The final velocity potential is then computed through an additive procedure allowed by the linearity assumption, such that

$$\phi = \sum_{i=1}^6 x_i \phi_i + H(\phi_0 + \phi_7) \quad (10)$$

The remaining steps leading to wave height distribution and wave forces on the structure are straightforward, as shown in Figure 5.

A detailed description of the typical Green Function Method, Garrison Model, is presented in Appendix A.

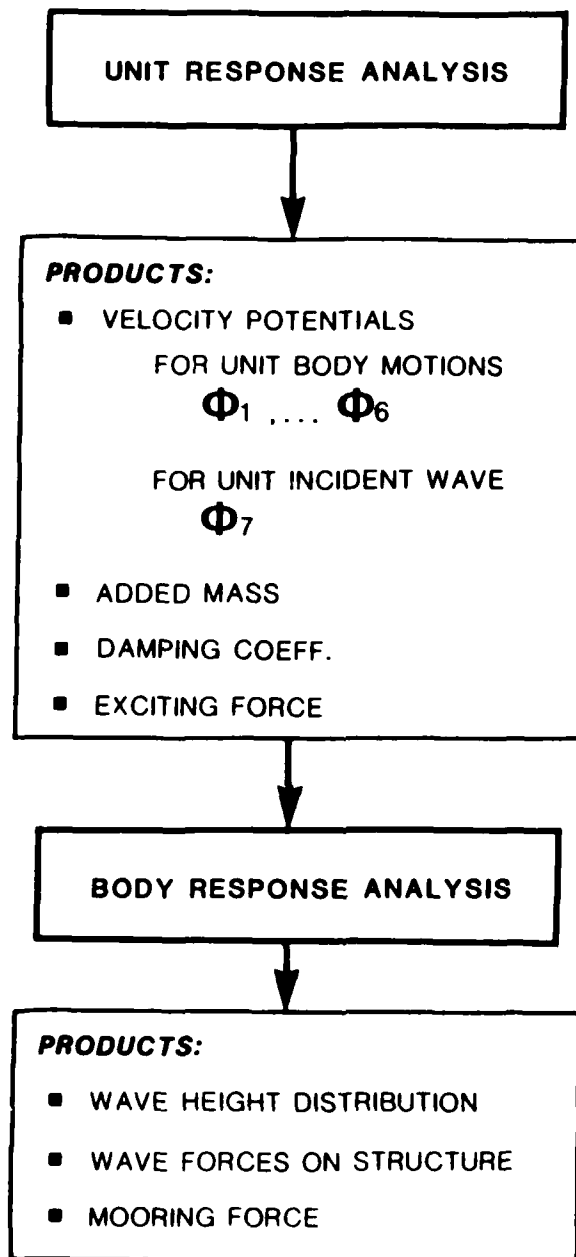


Figure 3. Flow of Computational Sequences in the Green Function Method (GFM) and Finite Element Method (FEM)

UNIT RESPONSE ANALYSIS

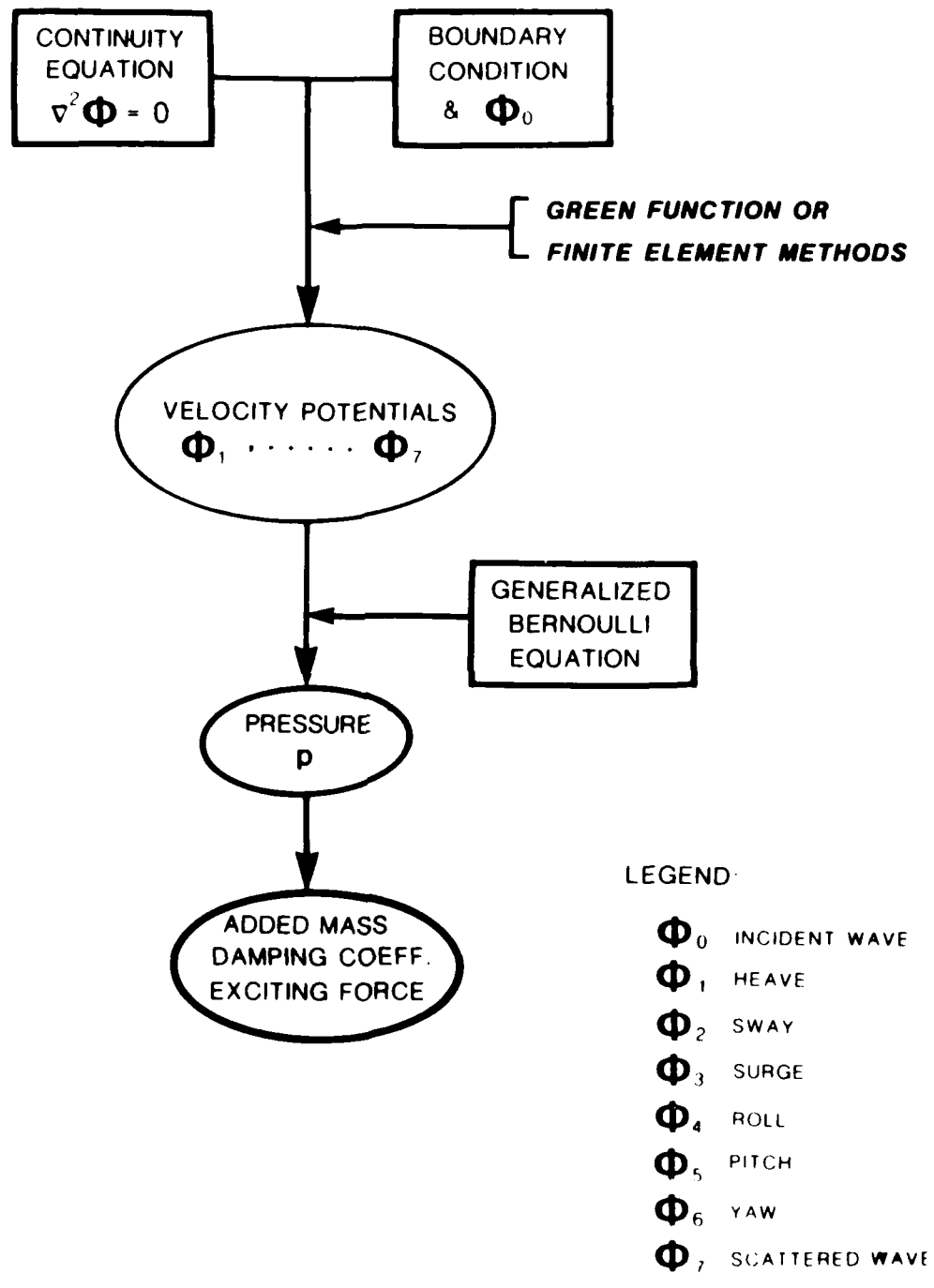


Figure 4. Flow Diagram for Unit Response Analysis in Green Function Method (GFM) and Finite Element Method (FEM)

BODY RESPONSE ANALYSIS

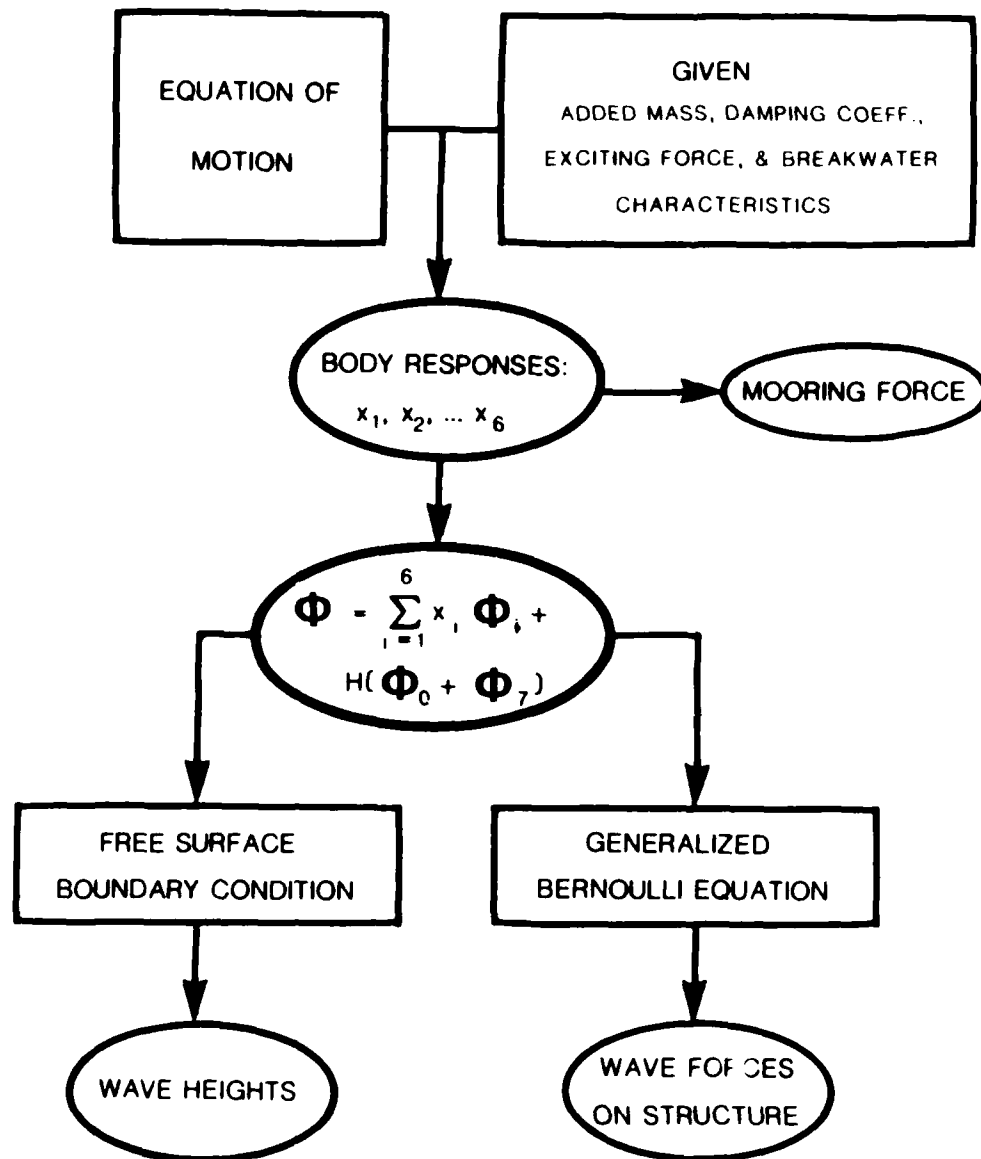


Figure 5. Flow Diagram for Body Response Analysis

3.4 Hybrid Green Function Method (HGFM)

This method combines partial features of EFEM and GFM, which facilitates computations dealing with complex breakwater geometry and variable water depths. In the HGFM, the domain of interest is partitioned into interior and exterior sub-regions, as in EFEM. The velocity potential in each sub-region is then expressed using both an Eigen Function series and a Green Function. The flow diagram in the HGFM is shown in Figure 6. In comparison with the flow diagram for the EFEM (Figure 1), it is clear that the flows of computational procedures in both methods are essentially the same.

Credit for the initial development of the hybrid approach belongs to Ijima and his associates. Their models handle three-dimensional cases with constant water depth (Ijima et al., 1975) and two-dimensional cases with variable water depths (Ijima et al., 1976). The hybrid model introduced by Kiyokawa et al., (1983) essentially boils down to a minor modification of the Ijima two-dimensional model allowing for easy determination of hydrodynamic coefficients (i.e., added mass, damping coefficient, and exciting force). A more detailed description of the Ijima model (1975) is presented in Appendix A.

3.5 Finite Element Method (FEM)

The flow of computational steps in the FEM is exactly the same as for the GFM, as shown in Figures 3, 4, and 5. The principal difference between the two models arises from the methods for determining velocity potentials under unit excitation in the unit response analysis. In the FEM, the continuity equation ($\nabla^2 \phi = 0$) is transformed into integral equations which are defined over the entire domain of the fluid, in contrast to the GFM in which the governing integral equations are defined only on the boundary. In the FEM, therefore, the entire domain is discretized into small

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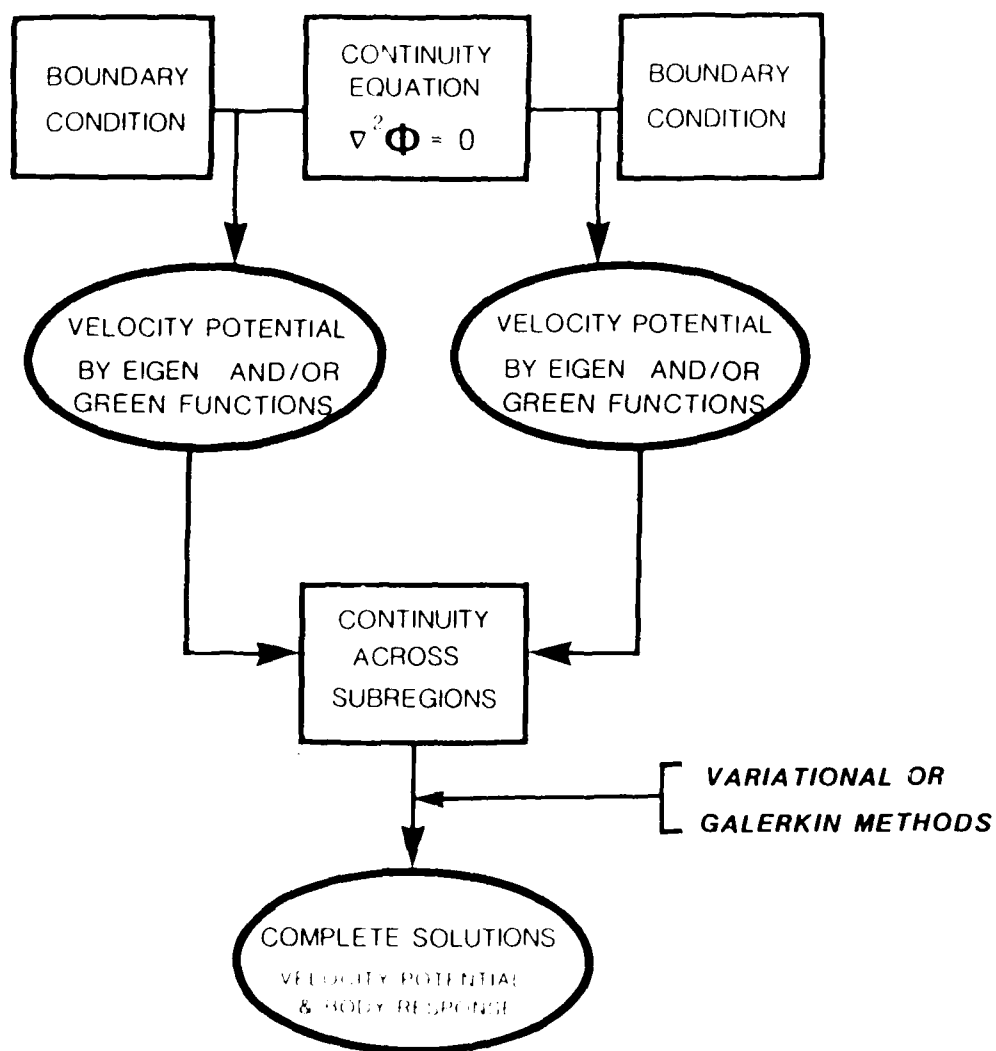


Figure 6. Flow Diagram for Hybrid Green Function Method (HGEM)

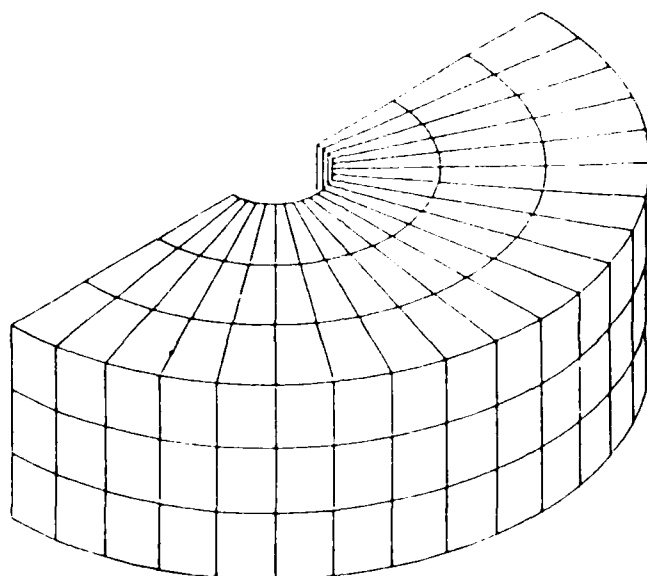
elements, while the discretization in the GFM applies only to the boundary, as shown in the examples in Figure 7.

The initial FEM model proposed by Zienkiewicz (1969) was designed to compute scattered waves around a fixed structure only. Hara (1978) expanded this model to allow for the analysis of floating bodies including both wave height distribution and forces on the structure. Bando et al., (1984a, b) introduced further improvements by incorporating computational accuracy and efficiency. Chen et al., (1974) introduced a simplified version of the FEM with a long wave assumption, and Yue et al., (1978) extended the long-wave model to three-dimensional problems.

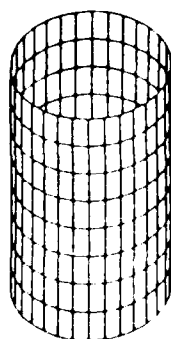
The accuracy and computing time in the FEM are sensitive to the way the radiation condition at infinity is treated. In order to formulate the radiation condition into the FEM, the behavior of the velocity potential in the far field should be expressed by a certain analytical form, since the entire fluid domain extending to infinity could not be covered by usual finite elements. The Yue et al., model uses an Eigen Function expansion for far field behavior, while the Bando et al., model adopts a simple decaying function. In this sense, both approximate methods may be termed the Hybrid Finite Element Method. In fact, Yue et al., (1978) call their model the "Hybrid-Element" method.

3.6 Other Methods

In all models of this category the potential is written as the sum of wave potentials with a singularity each satisfying all conditions except the body surface boundary condition. This boundary condition is then used to determine the unknown coefficients included in each potential. Since the mathematical derivation and numerical manipulation are complicated, it appears difficult to generalize this approach for scattering by a long



A. DISTRIBUTION OF ELEMENTS IN DISCRETIZED COMPUTATIONAL DOMAIN
FOR FLOATING CIRCULAR CYLINDER IN FEM



B. DISCRETIZED BOUNDARY OF FLOATING CYLINDER IN GFM

Figure 7. Examples of Discretization for FEM and GFM

cylinder in obliquely incident waves or for arbitrary three-dimensional bodies.

This method was initially developed by Ursell (1949), for the case of a half-submerged circular cylinder in calm water of infinite depth, then extended by others to various shapes and finite water depths. Wang (1966) applied the approach to heaving oscillations of a half submerged sphere in a finite water depth.

4. Random Wave Input

4.1 Discretization

The directional wave spectrum $S(f, \theta)$ is expressed as:

$$S(f, \theta) = S(f) \cdot G(f, \theta) \quad (11)$$

in which $S(f)$ is the frequency spectrum and $G(f, \theta)$ is the directional function, so that

$$S(f) = \int_{-\pi}^{\pi} S(f, \theta) d\theta, \quad (12)$$

and thus

$$\int_{-\pi}^{\pi} G(f, \theta) d\theta = 1 \quad (13)$$

For the purpose of numerical computation, Eq. (11) is discretized, such that

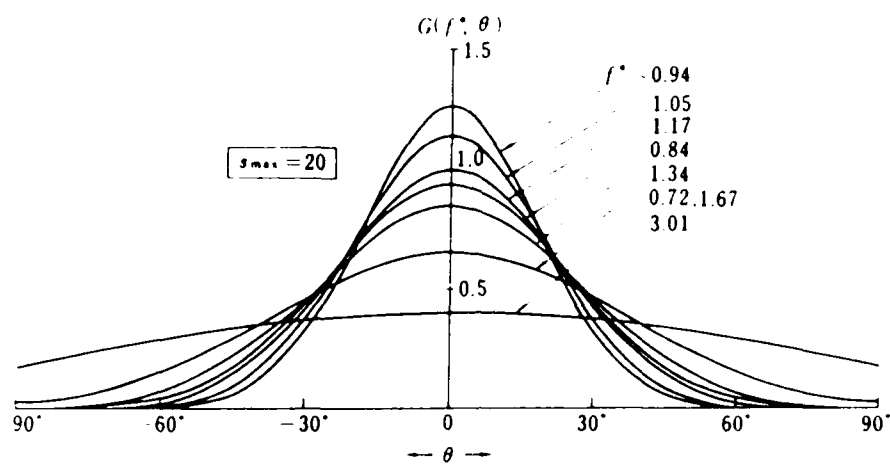
$$S(f, \theta) = \sum_{i=1}^N \left[\Delta S(f_i) \left(\sum_{j=1}^n G(f_i, \theta_j) \right) \right] \quad (14)$$

From the numerical point of view, decisions must be made on the optimal number (n) of discrete bands in the directional variability and that of the frequency domain (N). For instance, as the number of wave direction bands (n) is increased, the accuracy of the computation will improve, but the rate of improvement would diminish as the number n exceeds some critical value. Thus, there exists an optimum value for n where the accuracy and computational economy are best balanced. The same argument should apply to the optimal number of frequency bands, N.

4.2 Optimum Discretization of Wave Direction

Optimum discretization of wave direction has been investigated in a series of papers by Goda et al., (1975, '77), Nagai (1972 a, b, c), Nagai et al., (1974), and Takayama et al., (1978). These Japanese studies typically used the Mitsuyasu directional function (1981) with modifications introduced by Goda and Suzuki (1975).

The Mitsuyasu directional function contains the so-called wave spreading parameter S which relates the spreading characteristics to wave frequency. For instance, in any sea state, wave spreading is at its minimum near the peak frequency f_p . It is less in swell than in wind waves, and is progressively less with decay distance. Figure 8 shows the directional function by Mitsuyasu (1981).



Note: $f^* = f/f_p$

Figure 8. Examples of Mitsuyasu Directional Function for Spreading Parameter $S_{max} = 20$ (Goda, 1985).

The Mitsuyasu directional function is written as:

$$G(f, \theta) = \frac{1}{\pi} 2^{2S-1} \frac{(S!)^2}{(2S)!} \cos^{2S} \frac{\theta}{2} \quad (15)$$

in which

$$S = \begin{cases} S_{\max} \left(\frac{f}{f_p} \right)^{-2.5} & \text{for } f > f_p \\ S_{\max} \left(\frac{f}{f_p} \right)^5 & \text{for } f \leq f_p \end{cases} \quad (16)$$

and

$$f_p = \frac{1}{1.05 T_{1/3}} \quad (17)$$

according to Goda and Suzuki (1975). The term S_{\max} is the spreading parameter for the peak frequency $f = f_p$, and is expressed as

$$S_{\max} = 11.5 (2 \pi f_p U_{10}/g)^{-2.5} \quad (18)$$

in which U_{10} is the wind speed at a 10-m height above water.

In practical cases, S_{\max} assumes the following values (Goda and Suzuki, 1975; Goda, 1985):

$S_{\max} = 10$	Wind wave
$S_{\max} = 25$	Swell with a medium steepness after a short decay distance
$S_{\max} = 75$	Swell with a low steepness after a long decay distance

Discretization scheme must be adjusted to the S_{\max} values because the spreading characteristics vary greatly due to this parameter. For instance, when S_{\max} is large, one should not be concerned over the direction near $\pm \pi/2$. Takayama *et al.*, (1978) showed limiting wave directions θ_{\max} in the numerical computation to be considered as the function of S_{\max} as follows:

S_{\max}	θ_{\max}
10	89°
20	84°
30	76°
40	70°
50	64°
60	60°
70	56°
80	53°
90	51°
100	49°

Another well-known directional function is the SWOP (Stereo Wave Observation Program; see Kinsman, 1965) model. The wave spreading behavior in the SWOP model is frequency-dependent and is the function of a parameter related to the wind speed at a 5-m elevation, an ambiguous parameter. This ambiguity must be resolved before the SWOP model can be used for practical application.

In addition to a spreading parameter S_{\max} , there exists another parameter which influences the discretization of wave direction: the location of interest relative to the breakwater. This additional parameter is essentially a distance from the tip of the breakwater

$$R/L$$

in which R is the distance and L the local wave length.

Takayama et al., (1978) have shown that for any value of S_{\max} , the larger the parameter R/L , the larger must be the number of directional discretization in order to preserve an acceptable level of accuracy in the computation. In the case of a semi-infinite breakwater, these investigators proposed optimum values for n as a function of S_{\max} and R/L as shown in Table 2. Their results indicate that the number of directional discretization can be small if the location of interest lies close to the tip of the breakwater. However, for locations far from the breakwater tip, a greater number of discretization is necessary.

TABLE 2. OPTIMUM NUMBER OF DIRECTIONAL DISCRETIZATION
FOR WAVES INCIDENT TO SEMI-INFINITE BREAKWATER

R/L	S_{\max}			
	10	25	75	1000
5	8	8	6	6
10	18	15	12	9
20	20	19	16	10
50	25	23	20	12
80	30	25	23	13

Takayama et al., (1978) suggested a formula giving the optimum value of n as the function of S_{\max} and R/L as follows:

$$n = \{ 19.1 - 3.8 \log (S_{\max}) \} \log (R/L) \quad (19)$$

Figure 9 shows an example of computation of diffraction coefficients behind a semi-infinite breakwater for three directional bands: 5° ($n = 36$), 10° ($n = 18$), and 20° ($n = 9$). The principal direction of the wave was perpendicular to the breakwater. In this case, the directional discretization was obviously inadequate at $n = 9$ but too fine at $n = 36$. Using Eq. (19), the optimum value for n is determined as $n = 29.11 \sim 30$ (i.e., 6°) for, say, $R/L = 80$. For $R/L = 40$ (i.e. a position closer to the tip of the breakwater), one gets $n = 24.5 \sim 25$ (i.e. 7°).

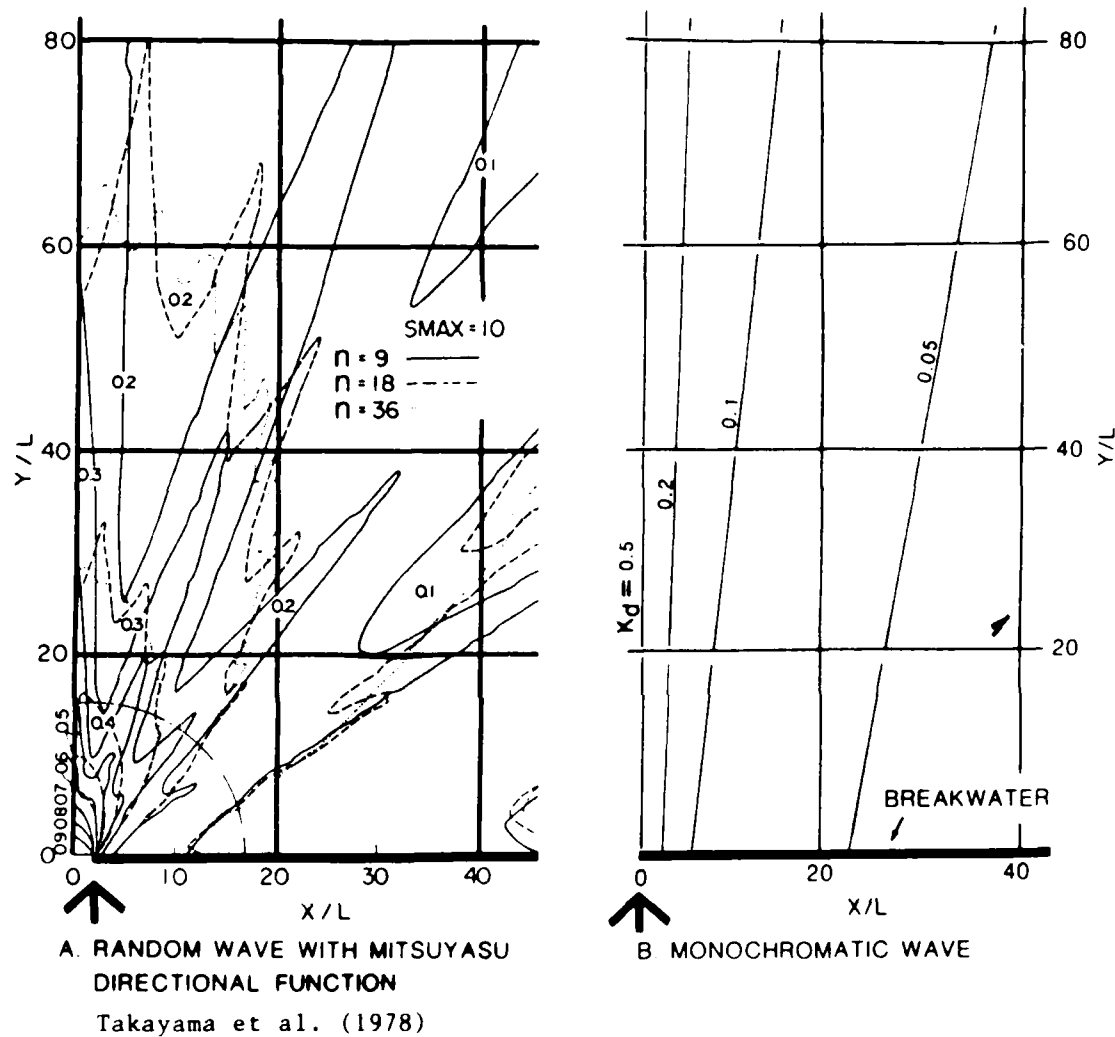


Figure 9. Contrasting Diffraction Patterns Due to Different Input Waves on Semi-Infinite Breakwater: A. Random Wave With Directional Spreading. B. Monochromatic Wave.

4.3 Discretization of Frequency Bands

Takayama et al. (1978) performed a numerical experiment in which fluctuations in the diffraction coefficient K_d were investigated as a function of the number of discretized frequency bands, N , in the case of a semi-infinite breakwater. The experiment, in which the directional discretization was held at $n = 36$, showed that the diffraction coefficients K_d remained essentially stable for values of N greater than 2. The diffraction coefficient was also found to fluctuate little with variations in S_{\max} and R/L . This meant that the computational accuracy was far less dependent on the discretization of frequency bands than that of the wave direction. In this experiment, for instance, if the computational stability was defined as

$$\left| \frac{K_d - K_d'}{K_d'} \right| = \epsilon \quad (20)$$

in which K_d and K_d' are diffraction coefficients for discretization N and N' ($N < N'$), respectively, N was as low as $N = 5$ to ensure

$$\epsilon < 2\% \quad (21)$$

Furthermore, Nagai (1972) recommends that the discretization of frequency bands must satisfy the following two conditions (Figure 10).

- 1^o The non-dimensional spectral energy contained in each discretized band should be the same:

$$\Delta E = \frac{1}{N} \int_0^{f_n^*} S(f^*) df^* \quad (22)$$

in which f_n^* is the non-dimensional cutoff frequency and $f^* = f/f_p$.

- 2^o The representative frequency f_c^* in each discretized frequency band should be determined in such a way that the second moment in each band around f_c^* is the same as the second moment relative to $f = 0$, i. e.,

$$f_c^{*2} \Delta E = \int_{f_{N-1}^*}^{f_N^*} f^{*2} S(f^*) df^* \quad (23)$$

in which

$$f_{N-1}^* < f_c^* < f_N^*$$

Nagai (1972) derived analytical expressions for ΔE and f_c^* in the case of Pierson-Moscowitz and Bretschneider spectra (see Figure 10).

In addition to these criteria, which lead to smaller discretized bands around the peak frequency, attention should also be paid to the resonance frequency of a floating body. The body response could grow sharply at the resonance frequency even though the incident wave energy at this frequency may be relatively small.

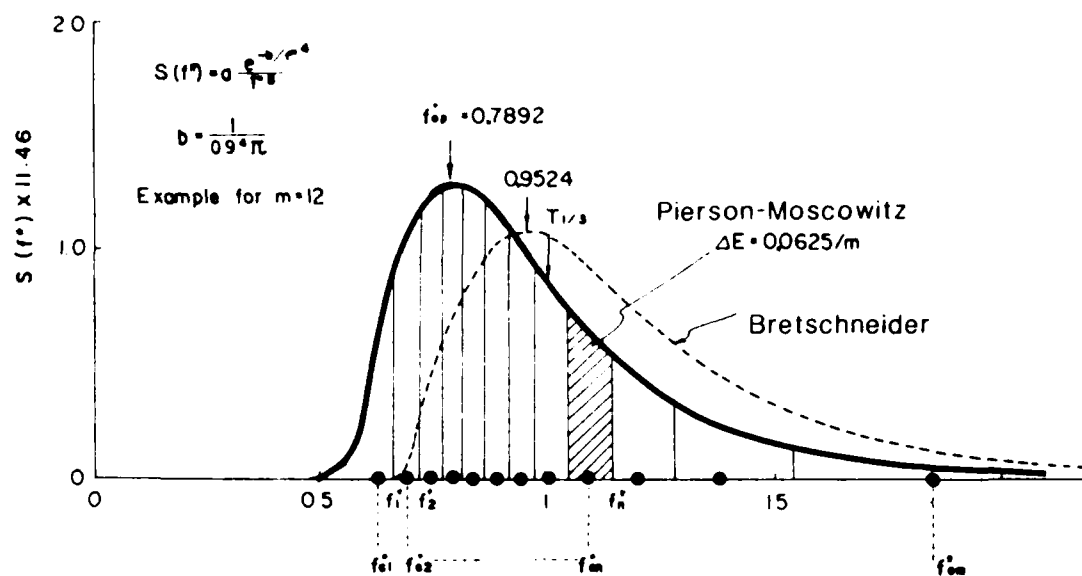


Figure 10. Example of Discretization of Frequency Bands (Nagai, 1972).

5. Evaluation of Models

5.1 Initial Screening

The comprehensive list shown in Table 1 was prepared to include numerical models dealing with floating bodies in general. Some of these models are not exactly applicable to floating breakwaters without substantial modifications and hence should be removed from further consideration in this study.

In the group of Eigen Function Expansion Method, all but Ijima et al., (1972a) and Ito and Chiba (1972) were removed. Models by Mei and Black (1969) and Garrett (1971) apply to a fixed floating body and those by Black et al., (1971) and Yeung (1981), only to an oscillating floating body in a calm sea. Another model, Ijima et al., (1972b) treats a vertical cylinder which obviously departs from the usual geometry of the floating breakwater. The model by Ijima et al., (1972a), which is retained in this group, deals with an elongated floating structure and hence represents a two-dimensional case of a floating breakwater with constant water depth. The model by Ito and Chiba (1972) is an abridged version of Ijima et al., (1972a), in which only the first terms of the infinite Eigen Function series were retained to achieve computational economy.

In the group of Green Function Method, there exist a number of three-dimensional Green Function models similar to those introduced by Kim (1965) and Garrison (1974, '78). But we decided to retain only the Garrison model inasmuch as the differences among them are small. The model by Georgiadis and Hartz (1982) was retained to enhance evaluation, because this model has the added capability of incorporating short-crested waves as input.

In the group of Hybrid Green Function Model, the model described by Yamamoto et al., (1980) is exactly the same as that of Ijima et al., (1976), and was therefore removed.

In the group of Finite Element Model, the models by Zienkiewicz et al., (1969, '77) and Hara (1978) were removed, because they are superceded by the more updated version described by Bando et al., (1984a, b). All the models in the category of "Others" were removed from further evaluation, because these models were intended for analysis of ship motions in the seaway, and were not completely applicable to a floating breakwater.

As a result of the screening process described above, the following ten models were retained for further evaluation:

Eigen Function Expansion Models:

Ijima et al., (1972a)
Ito and Chiba (1972)

Green Function Models:

Garrison (1974, '78)
Adee and Martin (1974)
Georgiadis and Hartz (1982)

Hybrid Green Function Models:

Ijima et al., (1975)
Ijima et al., (1976)
Kiyokawa et al., (1983)

Finite Element Models:

Yue et al., (1978)

Bando et al., (1984a,b)

5.2 Criteria for Evaluation

The models were evaluated against nine criteria:

. Accuracy	(15%)
. C.P.U. Time	(15%)
. Storage Requirement	(10%)
. Input Preparation	(15%)
. Diffraction Effect	(15%)
. Directional Wave	(10%)
. Breakwater Shape	(12%)
. Water Depths	(8%)
. Code Availability	(0%)

The numbers within the parentheses represent the percent weight assigned to each criteria.

We considered that the computational performance of the model should be given more than 50% of the overall significance. Accordingly, we assigned a weight of 55% to the first four criteria: accuracy, C.P.U. (central processing unit) time, computer storage requirements, and the work required to generate the input data (particularly the work required for discretization). The functional attributes of the model, consisting of the capabilities to incorporate three-dimensional effects (i.e., diffraction processes and directional wave spectrum for the input wave) and the flexibility to accommodate various breakwater shapes and water depths, were assigned a combined total weight of 45%. The code availability was not

assigned a weight because it was safe to assume that the code would be available in any case.

The tally of the scores for each screened model is summarized in Table 3.

5.3 Quantitative Evaluation

Regarding the accuracy of computation, all the models with the exception of Ito and Chiba (1972) and Georgiadis and Hartz (1982), were given the maximum score of 15%. The model by Ito and Chiba (1972) employed only the first term of an infinite Eigen Function series for velocity potential in the Ijima et al., (1972a) model. Although this was an approximate solution, they reported fairly close agreement between numerical results and the experimental data.

In the model by Georgiadis and Hartz (1984), the radiation condition which is only valid at infinity is prescribed on the artificial boundary at a finite distance from the breakwater. Although this artifact undoubtedly provides computational expediency, the influence of this approach on the accuracy of the results is unknown and should be examined.

The criteria of computing time (C.P.U. time), computer storage requirements and input data generation relate to computational economy. The three-dimensional FEM (Yue et al., 1978; Bando et al., 1984a, b) models and the three-dimensional Green Function models (Garrison 1974, '78) scored poorly with respect to computational economy. This is mainly because these models require discretization in the three-dimensional domain of interest, usually a labor-intensive effort. Out of the maximum possible score of 40% for computational economy, the FEM models by Yue et al., (1978) and Bando et al., (1984a, b) were given only 12% each, and the GFM model by Garrison (1974, '78) only 18%. The three-dimensional Hybrid Green

TABLE 3. QUANTITATIVE EVALUATION OF SCREENED MODELS

MODELS		SCORES (%)									
		COMPUTATIONAL CRITERIA (55%)				FUNCTIONAL CRITERIA (45%)				CODE AVAILABILITY	TOTAL
		ACCURACY (15)	CPU (15)	STORAGE REQUIREMENTS (10)	INPUT GENERATION (15)	DIFFRAC- TION EFFECT (15)	DIREC- TIONAL WAVE (10)	BREAK- WATER SHAPE (12)	WATER DEPTH (8)		
EFPM	4. Iijima et al. (1972a)	15	13	9	15	0	0	6	5	YES	63
	6. Ito & Chiba (1972)	11	15	10	15	0	0	6	5	YES	62
	9. *Garrison (1974, 78)	15	5	5	8	15	10	12	5	YES	75
GFM	10. Adee & Martin (1974)	15	7	8	15	0	0	8	3	YES	56
	11. Georgiadis & Hartz (1982)	12	12	8	14	0	10	8	8	YES	72
	12. *Iijima et al. (1975)	15	11	7	11	15	10	10	5	YES	84
HGFM	13. Iijima et al. (1976)	15	12	8	14	0	0	8	8	YES	65
	15. Kiyokawa et al. (1983)	15	12	8	14	0	0	8	8	YES	65
	18. *Yue et al. (1978)	15	3	3	6	15	10	12	8	YES	72
FEM	20. *Bardo et al. (1984a, b)	15	3	3	6	15	10	12	8	YES	72

EFPM: Eigen Function Expansion Method GFM: Green Function Method
 HGFM: Hybrid Green Function Method FEM: Finite Element Method

Note: 3-dimensional models marked with (*).

Function Model (Ijima et al., 1975) scored 29%, demonstrating a considerable advantage over the three-dimensional FEM and GFM models. In fact, two-dimensional models scored 30% or above on computational economy for obvious reasons.

In Table 3, all the three-dimensional models, marked *, possess the ability to accommodate directional wave spectra and diffraction effect around the edge of the breakwater. The model by Georgiadis and Hartz (1982) is not exactly a three-dimensional model, since it cannot handle diffraction effect, although it can accommodate the short-crested oblique waves. This model deals with an infinitely long floating structure, essentially a two-dimensional approach.

The models using the Eigen Function Expansion Method have limitations in terms of accommodating versatile breakwater geometry. The hybrid model by Ijima et al., (1975) dealt with cylinders with elliptical and rectangular sections, which can be elongated into a horizontal dimension to simulate the usual oblong shape of the floating breakwater. The FEM and GFM models typically have the ability to accommodate arbitrary three-dimensional breakwater geometry.

Whereas the FEM models can handle variable water depths, the GFM models cannot because the Green Function is usually derived for a constant water depth. However, this typical limitation of the GFM model can be overcome through modifications such as those incorporated by Georgiadis and Hartz (1982), Ijima et al., (1976) and Kiyokawa et al., (1983). The model by Adey and Martin (1974) only applies to infinite water depth, hence is not appropriate to a finite depth where the floating breakwater is installed.

5.4 Examples of Computational Speed

In order to gain a further understanding in computational time, a comparison of the CPU time among the models has been made. Information on the performance of six programs based on the following numerical models was obtained. The results of the Yue et al., model were found in their paper (1978). The others were obtained through private communication with Mr. Takahashi of the Kajima Corporation.

Eigen Function Expansion Models:

Ijima et al., (1972a)

Ito and Chiba (1972)

Green Function Models:

Garrison (1974, '78)

Finite Element Models:

Yue et al., (1978)

Bando et al., (1984a, b)

To arrive at a standard of comparison, the response of a floating dock in a simple harmonic wave field was computed by each program (see Figure 11). Table 4 summarizes the CPU time per single wave frequency as a function of the number of unknowns solved by each model. The number of unknowns is essentially associated with the number of simultaneous equations to be solved, and in the case of the GFM and the FEM, it is further related to the number of discretized elements.

The computer used by Takahashi was a Hitachi M-280H, considered equivalent to an IBM 3081K. The computer used by Yue et al., (1978) is apparently a main-frame variety, but its exact model is unknown.

Both programs based on the EFEM demonstrated the remarkable computational efficiency, showing a faster CPU time by 1 to 3 orders of magnitude. The discrepancy in CPU time between the two GFM programs is attributed to the different numerical evaluation methods for the Green Function. The second GFM program is expected to contain an advanced subroutine which speeds up the evaluation of Green Function.

Using the FEM model by Bando et al., computations for the different numbers of unknowns resulted in widely varying CPU times as shown. As a rule of thumb, the CPU time in the FEM is proportional to the square of the number of unknowns. In this case, where the number of unknowns is reduced to a practical minimum of 400, the CPU time is estimated to achieve the level comparable to that of the GFM.

Figure 12 shows the body response computed by the Ito and Chiba and the Bando et al., models. Agreement between the two-dimensional and the three-dimensional models is fairly close in terms of body response.

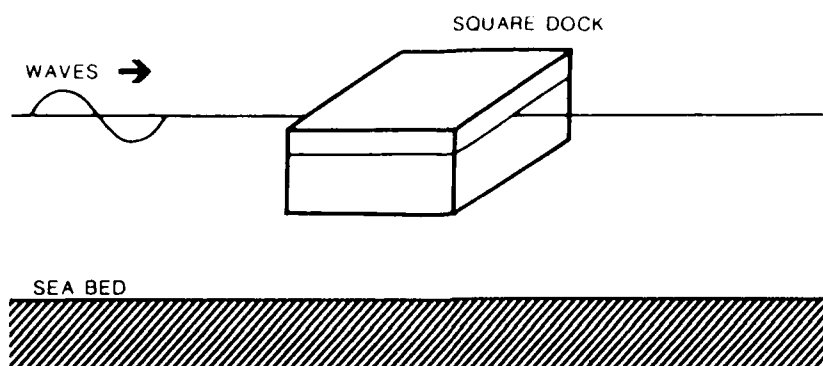


Figure 11. A Floating Square Dock in Waves

TABLE 4. EXAMPLES OF COMPUTATIONAL SPEED

METHOD	MODEL # IN TABLE 3	CPU (sec)	NO. OF UNKNOWN
EFEM (Ito)*a	6	0.005	5
EFEM (Ijima)	4	1	3 + ?
GFM (Garrison)*b	9	100	45
GFM*c	9	11	45
FEM (Bando)	20	1150	3893
FEM (Bando)	20	300	2165
FEM (Bando)	20	10 - 20*d	400
FEM (Yue)*e	18	200*f	435

*a : Written by Bando

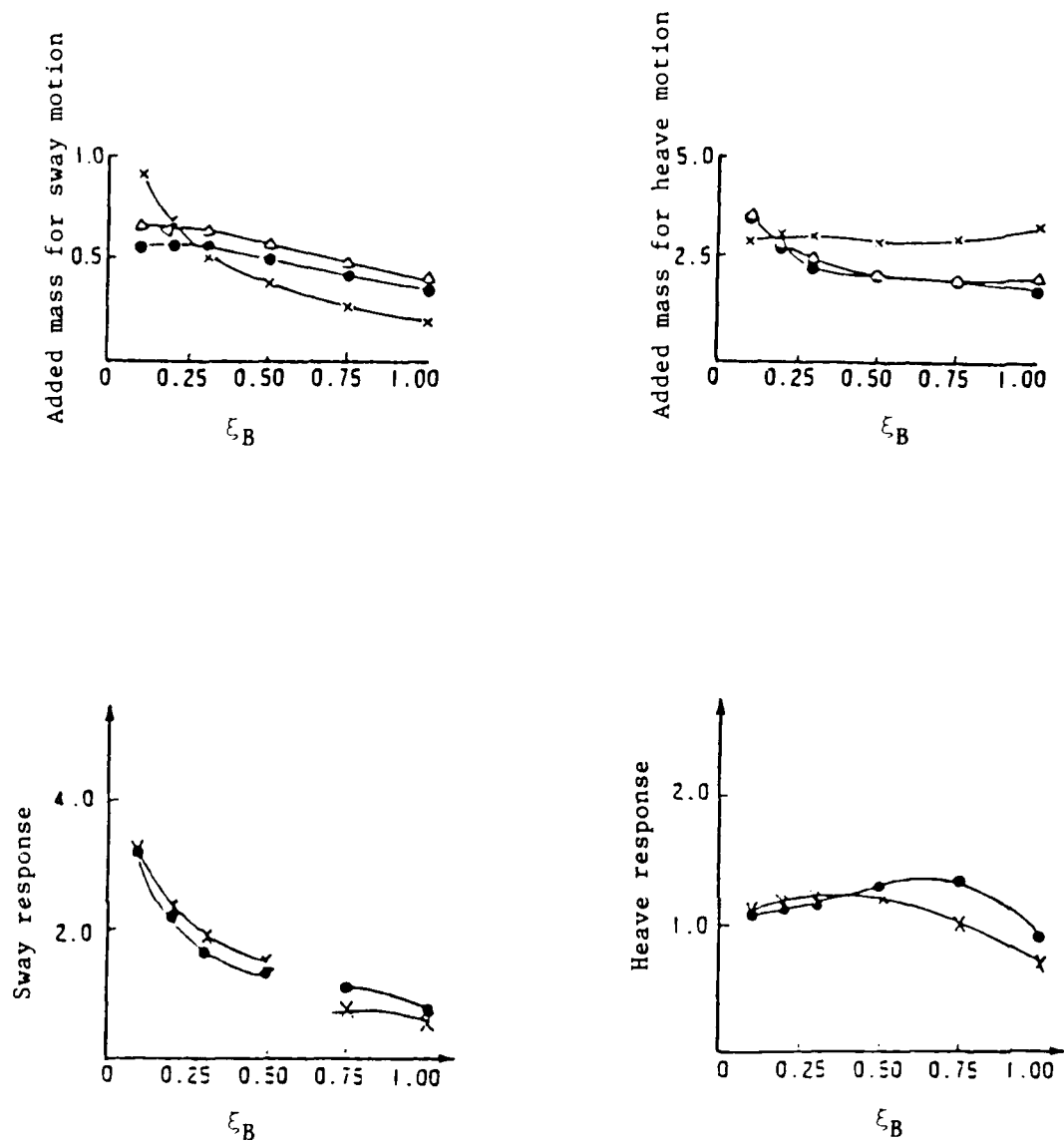
*b : Written by Bando

*c : Commercial package sold by Veritec

*d : Estimation

*e : Exciting force only

*f : Computed in 1978



LEGEND:

x—x : Ito and Chiba model (EFEM)

•—• : Bando et al. model (FEM)

ξ_B : Dimensionless frequency parameter

Figure 12. Examples of the Response of a Floating Dock Computed by the EFEM and the FEM (Takahashi, 1985)

5.5 Costs

Program cost is one of the most important parameters in the selection of the best numerical model for CERC. The following Table summarizes the cost information obtained through communications with Professor Ijima of Kyushu University, Professor Newman of MIT, the MIT Patent Office, Dr. Garrison, and the Kajima Corporation in Japan.

TABLE 5. COSTS OF VARIOUS PROGRAMS

PROGRAM	AUTHOR OR OWNER	COST	DELIVERY FORM
EFEM (6)	Kajima Corp.	\$1,000	Source Code
HGFM (12)	Ijima, T.	\$4,000-\$5,000	Source Code
GFM (9)	Garrison, C. J.	\$35,000	Source Code
GFM (9)	Kajima Corp.	\$6,000	Source Code
FINGREEN	Newman, J. N.	\$200	Object Module
FINGREEN	Newman, J. N.	\$10,000	Source Code

The number within the parantheses denotes the model number from Table 3.

The Eigen Function Expansion Method (6) based on the Ito and Chiba two-dimensional model, was written by Bando of Kajima. Although the license fee for the GFM (9) program owned by Garrison is \$20,000, a higher fee has been set for CERC because the program is expected to be used by quite a large number of people. Kajima's GFM (9) program was written by Bando after the original paper by Garrison. Governmental agencies can obtain the object module of the

program "FINGREEN" for a handling fee of only \$200, but must pay \$10,000 for the source code.

6. Recommendations

6.1 Discussion

The Hybrid Green Function Model by Ijima et al., (1975) scored well in all the criteria tested, achieving the highest tally of 84%. It also outscored the second-ranking model by Garrison (1974, '78) by a clear margin of 9%. Comparing these two high-ranking models, we find that the Ijima et al., (1975) model excels in computational economy (29% versus 18%), while the Garrison (1974, '78) model is somewhat superior in terms of the ability to accommodate breakwater shapes (12% versus 10%).

Recently, an efficient algorithm was developed (Newman, 1985) to expedite the computational speed in the evaluation of the Green Function. Packaged into a subroutine named "FINGREEN", this method replaces the numerical evaluation of the relevant integrals with multidimensional approximations in economized polynomials. A substantially faster computational speed is said to be achieved by this method than by conventional methods based on direct numerical integration. We have not had the opportunity of testing whether or not the computational efficiency of the Garrison model, when coupled with "FINGREEN", would exceed that of the Ijima et al., (1975) model. A more detailed description of "FINGREEN" is presented in Appendix B.

Three other models, those by Georgiadis and Hartz (1982), Yue et al., (1978) and Bando et al., (1984a, b), scored an identical 72%, ranking slightly behind the Garrison model which scored 75%. Of these three third-ranking models, the Georgiadis and Hartz (1982) model is not strictly three-dimensional, since it cannot incorporate

the wave diffraction effect. The models by Yue et al., (1978) and Bando et al., (1984a, b) are very similar to each other.

The mooring force is one of the most important sources of information in the design of floating breakwaters. Although this force is computed automatically in the model, the numerical result is highly dependent on the spring constant given by the users. The spring constant, which is the gradient of the displacement-force curve, depends on the magnitude of its displacement, hence varies continuously throughout the motion of the floating body. Figure 13 shows an example of this non-linear relationship in the case of a single catenary-shaped mooring cable. A practical idea is to pre-select a spring constant by anticipating the range in which the displacement would occur. Another problem is the conjugation of mooring forces in a multiple cable system. To date, little is known as to how to distribute the total computed mooring force to individual cables.

Presently, state-of-the-art models with floating breakwater applications are continuously being updated and improved. The preceding discussion was based on observations made during the immediate time this review was performed.

6.2 Recommendations

We conclude that the Hybrid Green Function Model by Ijima et al., (1975) has the best overall attributes. It not only achieved the highest total score with a clear margin over the next best model, it also scored evenly on all the criteria tested. The Ijima et al., (1975) model has been developed as a research tool, and is not in a user-friendly form. To make the program fit for routine use, refinement of program code, preparation of program documentation and a quality control review should be carried out. Prof. Ijima of Kyushu University, the co-author of the model, expressed willingness to cooperate in these tasks. The Ijima et al., (1975) model can be operated on a minicomputer.

For the purpose of obtaining a quick diagnostic assessment before committing a sophisticated three-dimensional analysis, the two-dimensional model by Ito and Chiba deserves special attention. This model can be operated on a microcomputer and is extremely inexpensive.

Ongoing improvements of state-of-the-art numerical modeling tools require users to seek information concerning the most recent advancements. Future models will be improved and related literature will be continuously updated.

In summary, the following recommendations are made at this time:

- 1) Acquire the Ijima et al., (1975) model and adapt it into a user-friendly form. The program itself is available at no cost, but the tasks for necessary adaption would cost \$4,000 to \$5,000.
- 2) Conduct test runs of the Ijima et al. model against selected scenarios of prototype tests by CERC and the Japanese Ministry of Transport. In these test runs, random waves with directional variability shall be used as input waves.
- 3) The two-dimensional Ito and Chiba model is worth consideration as a tool for speedy diagnostic assessment.
- 4) If additional financial resources are available, the combination of Garrison's GFM model and the "FINGREEN" subroutine may be worth consideration.
- 5) Efforts to improve the computation of the mooring force shall be made as part of the test runs by paying special attention to the existing field data.

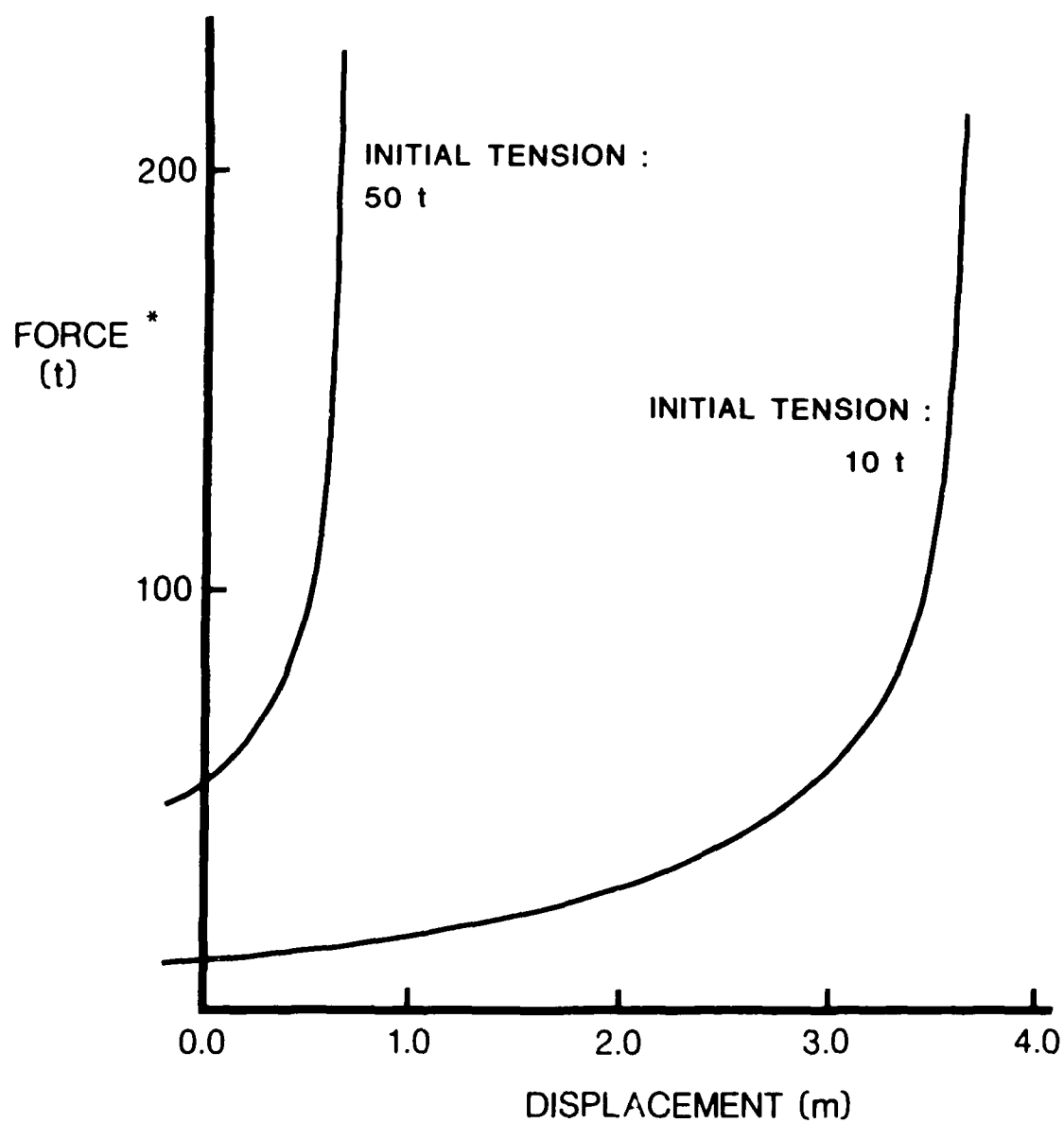


FIGURE 13. Force-Displacement Relationship in the Catenary Mooring Cable.

* - To obtain kilonewtons, multiply tons (2000 pounds force) by 8.8964

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Appendix A:

Detailed Description of Selected Models

Ijima et al., (1975) Model (Hybrid Green Function Model)

In this method the domain of interest is divided into two sub-regions of simple geometry as shown in Figure 2: the "interior" sub-region beneath the floating body bounded by the body, the sea bottom and the artificial borders, and the "exterior" sub-region with the free surface extending to infinity.

In each of the sub-regions, the velocity potential can be represented by the horizontal distribution functions (Green Function) and the Eigen Functions in the vertical direction. The velocity potential in the exterior sub-region satisfies any conditions other than the boundary conditions at infinity and at the artificial border, while the velocity potential in the interior sub-region, including some components of body motion, satisfies all the boundary conditions but the one at the artificial border. They are written in the following form:

$$\begin{aligned} \phi_E(x, y, z) = & \frac{gH}{2\sigma} \left[\{f_0(x, y) + f_1(x, y)\} \frac{\cosh k(z+h)}{\cosh kh} \right. \\ & \left. + \sum_{n=1}^{\infty} f_2^{(n)}(x, y) \frac{\cos k_n(z+h)}{\cos k_n h} \right] \end{aligned} \quad (A-1)$$

$$\begin{aligned}
\phi_I(x, y, z) = & \frac{gH}{2\sigma} \left[\psi_0(x, y) + \sum_{s=1}^{\infty} \psi_s(x, y) \cos \bar{s}(z+qh) \right. \\
& + \frac{1}{q} \frac{\sigma^2 h}{g} \left\{ \left(-\frac{\zeta}{H} - \frac{\omega_1 y}{H} + \frac{\omega_2 x}{H} \right) \left(1 + \frac{z}{H} \right)^2 \right. \\
& \left. \left. + \frac{1}{4} \left(\frac{2\zeta}{H} + \frac{\omega_1 y}{H} - \frac{\omega_2 x}{H} \right) \left(\frac{x^2 + y^2}{h^2} \right) \right\} \right] \quad (A-2)
\end{aligned}$$

where

$$kh \tanh(kh) = -k_n h \tan(k_n h) = \sigma^2 h / g$$

$$f_0(x, y) = -i \exp \{-ik(x \cos \omega + y \sin \omega)\}$$

$$\bar{q} = 1 - q$$

$$\bar{s} = s\pi/qh$$

The parameters $H, \sigma, h, qh, \zeta, \omega_1$ and ω_2 denote wave height and frequency of incident waves, water depth, draft, and amplitudes of body translation in the vertical (z) direction and rotations around x and y axes.

The unknown functions f_1, f_2, ψ_0 and ψ_s included in the velocity potentials satisfy the Helmholtz equation and can be expressed respectively by the Green Functions:

$$H_0^{(1)}(kr), K_0(kr), \log(1/kr) \text{ and } K_0(\bar{s}r)$$

When the Green's theorem is applied to each sub-region and the

Green Functions and the proper boundary conditions at the artificial border are substituted into the theorem, the equations defining the velocity potentials at the border are obtained. Solving these equations along with the body motion equations leads to the determination of the velocity potential at the border, which, in turn, can provide the value of velocity potential at any position in the region of interest.

This model features the decomposition of the velocity potential, which allows the adoption of simple form Green Functions and consequently results in economical computation. This feature also contributes to the easy input data generation since discretization is required only at the horizontal outline of the body because the vertical distribution functions, i.e. Eigen Functions, are separated from the velocity potential.

Garrison Model (GFM)

In this method the Green Function is used which satisfies the Poisson equation and all the boundary conditions but the one at the body surface.

$$\nabla^2 G(x, \xi) = -\delta(x, \xi) \quad (A-3)$$

$$\frac{\partial G(x, \xi)}{\partial z} - \frac{\sigma^2}{g} G = 0 \quad \text{at free surface} \quad (A-4)$$

$$\frac{\partial G(x, \xi)}{\partial z} = 0 \quad \text{at sea bottom} \quad (A-5)$$

$$\sqrt{r} \left(\frac{\partial G}{\partial r} - ikG \right) = 0 \quad \text{at infinity} \quad (A-6)$$

The particular expression for the Green Function appropriate to our boundary value problem is given in two different forms. One of these is the integral form:

$$G = \frac{1}{R} + \frac{1}{R'}$$

$$+ 2 \text{ P.V. } \int_0^\infty \frac{(\mu + \nu) e^{-\mu h} \cosh[\mu(\zeta + h)] \cosh[\mu(z + h)]}{\mu \sinh(\mu h) - \nu \cosh(\mu h)} J_0(\mu r) d\mu$$

$$+ 2 \pi i \frac{(k^2 - \nu^2) \cosh[k(\zeta + h)] \cosh[k(z + h)]}{k^2 h - \nu^2 h + \nu} J_0(kr) \quad (\text{A-7})$$

where

$$R^2 = (x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2$$

$$R'^2 = (x - \xi)^2 + (y - \eta)^2 + (z + 2h + \zeta)^2$$

$$r^2 = (x - \xi)^2 + (y - \eta)^2$$

$$\nu = \frac{\sigma^2}{g} = k \tanh(kh)$$

P.V. denotes the principal value of the integral. An alternate series form of the Green function is given by:

$$G = \frac{2\pi(\nu^2 - k^2)}{k^2 h - \nu^2 h + \nu} \cosh[k(\zeta + h)] \cosh[k(z + h)] [Y_0(kr) - iJ_0(kr)]$$

$$+ 4 \sum_{k=1}^{\infty} \frac{\mu_k^2 + \nu^2}{\mu_k^2 h + \nu^2 h - \nu} \cos[\mu_k(z + h)] \cos[\mu_k(\zeta + h)] K_0(\mu_k r) \quad (\text{A-8})$$

where

$$\mu_k \tan(\mu_k h) + v = 0$$

Comparing the Green Functions used in the Ijima and Garrison models it is readily apparent that the evaluation of the Green function in the Garrison model is much more difficult and time consuming.

In the Garrison model the hydrodynamic problem is first solved and then the body response is analyzed. We will discuss this two-step procedure here. For simplicity let us consider the case of the one-freedom-motion system. The equation of motion may be written in the following form:

$$\begin{aligned} m \frac{d^2 \alpha}{dt^2} + k \alpha &= \rho \int_{S_0} \frac{\partial \phi}{\partial t} n ds \\ &= -i \sigma \rho \int_{S_0} \phi n ds \end{aligned} \quad (A-9)$$

where α , m , k , ϕ , and S_0 are the displacement, mass, stiffness due to hydrostatic and mooring forces, velocity potential and the equilibrium immersed body surface, respectively. The right hand side of the equation represents the hydrodynamic force, which is only the drive force in this problem. The velocity potential ϕ can be split into three components in this case; ϕ_0 associated with the incident wave of unit wave height, ϕ_1 associated with unit body motion in calm water, and ϕ_2 associated with the scattering of the incident wave by the restrained body.

Because the velocity potentials due to incident and scattered waves are proportional to the incident wave height and the potential due to radiated waves ϕ_1 is linearly related to the amplitude of body motion,

the total velocity potential ϕ can be given by:

$$\phi = H \phi_0 + \alpha \phi_1 + H \phi_2 \quad (A-10)$$

Substituting this expression into equation (A-9), we obtain:

$$\begin{aligned} m \frac{d^2 \alpha}{dt^2} + k \alpha &= -i \rho \omega H \int_{S_0} (\phi_0 + \phi_2) n ds - i \rho \omega \alpha \int_{S_0} \phi_1 n ds \\ &= F_e + F_f \end{aligned} \quad (A-11)$$

where F_e is the force component due to the incident and scattered velocity potentials and F_f is the component due to radiated velocity potential generated by body motion. The force F_f , which is proportional to the amplitude of body motion, may be conveniently decomposed into two components in phase with the velocity and the acceleration and may be written in the following form:

$$F_f = -m^* \frac{d^2 \alpha}{dt^2} - c^* \frac{d \alpha}{dt} \quad (A-12)$$

where the coefficients m^* and c^* are termed the added mass and damping coefficients, respectively, since they assume corresponding roles in the equation of motion. Thus, using the definition of the hydrodynamic coefficients and rearranging terms, the equation of motion can be written

in the form:

$$(m + m^*) \frac{d^2\alpha}{dt^2} + c^* \frac{d\alpha}{dt} + k\alpha = F_e \quad (A-13)$$

In the above equation, m^* , c^* and the exciting force F_e may be evaluated prior to solving the equation, since all of them are independent of the body motion as discussed above.

Thus, in the first stage, velocity potentials due to the scattered waves by a fixed body and the radiated waves by the unit body motion in each direction are computed, and these potentials give the hydrodynamic coefficients m^* , c^* , and F_e . The equation of motion, with all coefficients defined, is then solved.

Appendix B

Subroutine FINGREEN

(Extracted From "USER MANUAL FOR 'FINGREEN'"

By J. N. Newman and P. D. Sclavounos)

1. INTRODUCTION

FINGREEN is a file of subroutines intended for use in computing the velocity potential and its derivatives for the 3-dimensional free surface source of harmonic time dependence, in a fluid of arbitrary depth, subject to the linearized free-surface condition. Normally this file will be used in conjunction with a radiation/diffraction main program provided by the user, which computes the hydrodynamic pressures and forces acting upon a floating or submerged vessel.

FINGREEN can be used to evaluate either the infinite or finite-depth source potential. No practical limits restrict the magnitude of the depth if it is finite, nor is there any degradation of accuracy if the depth is large. This presents the user with a choice in solving problems in relatively deep water, and comparisons between the infinite-and finite-depth options may be made to determine the effect of finite depth. Except for a saving in computational time, there is no advantage to using the infinite-depth option as an approximation for large but finite depths. The finite-depth option is recommended in special cases involving wave interaction with very deep structures in close proximity to the bottom, where the fluid is deep in relation to the wavelength but not in relation to the body configuration.

The FINGREEN file consists of a primary subroutine FGREEN and two subsidiary subroutines IGREEN and HGREEN which are called from and return

to FGREEN. This file is in conventional FORTRAN 77 source code, and is to be linked with the main program following compilation. Usage follows the FORTRAN convention, with a call from and return to the main program. Single-precision real numbers are used throughout the subroutine. The object code file requires 17408 bytes of storage on the VAX 11/780.

This manual describes the use of FINGREEN, provides check values of the output, and briefly describes the various parts of the code. Reference 1 may be consulted for a description of the algorithms which are used, and for definitions of the source potential in infinite and finite depths (equations 1-3 and equation 8, respectively, in Reference 1).

2. ACCURACY

FINGREEN has been designed to yield an accuracy of six decimal places for the source potential and its first derivatives, and five decimal places for the second derivatives. Here the designation of six decimal places accuracy is used to denote either the absolute or relative error, whichever of these two is smaller. If the output is less than the unity in absolute value the absolute error should be less than $1\text{E-}06$ ($1\text{E-}05$ for the second derivatives) whereas if the output is greater than unity the relative error should be smaller than the same amount. This accuracy test is applied separately to each real component of the output array.

The principal qualification to this general statement is for the regime $H < 1$, where the dominant length scale governing the nondimensionalized source potential and its derivatives is the depth parameter h , instead of the inverse wavenumber $1/K$. For this reason, the design objective in the regime $H < 1$ is modified to preserve six decimals accuracy (five for the second derivatives) for the products of H with the potential, H with the first derivatives, and H with the second derivatives.

In some instances, the accuracy may be degraded by the accumulation of round-off errors and/or subtractive cancellation. One specific and obvious example of subtractive cancellation occurs when the source and field points are very close to the bottom of the fluid domain, and to each other; in this case the image source beneath the bottom is dominant (the second term on the right side of equation 10a in Reference 1), and (since the origin is at the free surface) the vertical component ($2h + z + \zeta$) involves the difference between nearly-equal quantities.

A similar problem occurs for large values of X , since the oscillatory phase of the output is limited in precision. An estimate of this error is the product of 2π multiplied by the maximum error in the machine evaluations of $\sin(X*ROH)$ and $\cos(X*ROH)$. Here ROH denotes the nondimensional wavenumber corresponding to the depth H . In the finite depth case this parameter is limited in precision to a minimum of 6 significant figures by the approximation of the wavenumber as the root of the dispersion relation. Thus the maximum accuracy of the outputs is degraded by approximately one decimal place if $X=10$, or two decimal places if $X=100$. In the infinite-depth limit (or for values of $H>10$, where the error in the approximation of ROH is negligible) the limiting factor is the accuracy of the machine evaluation of the trigonometric functions of large argument X , which is of similar magnitude to the above estimates for computing systems with 6-7 decimal single-precision accuracy.

Since the actual errors are machine-specific, users who are concerned to achieve extreme accuracy are advised to perform their own tests by comparing the outputs from FINGREEN with a double-precision table. Appendix 1 (Appendix not included) contains tables of test outputs both from FINGREEN (run on the VAX 11/780 using single-precision arithmetic) and from a special double-precision program DGREEN which is designed to be accurate to twelve decimals. For the infinite-depth case double-precision tables of FR and $FR1$ are contained in Reference 2.

3. ESTIMATES

The running times of FINGREEN depend on the inputs, and the type of computing system which is used. The following estimates are based on use of the VAX 11/780 system. Limited tests on an IBM 370/168 and an IBM 370/3033N indicate that the time required on each of these systems is about 40% and 15% respectively of those on the VAX. In other words, one call to FINGREEN for the infinite depth case which may require 1.0 milliseconds on the VAX will require 0.4 milliseconds on the 370/168, or 0.15 milliseconds on the 370/3033N. The same run on the IBM PC microcomputer equipped with a floating point coprocessor requires about 15 milliseconds.

For the infinite-depth case, the time depends on the two coordinates X, Y, where $X=X_1$ has the same meaning as in finite depth and Y is the sum of the two vertical coordinates, $Y=X_3+X_6$. The maximum run times will occur when X and Y are of comparable magnitude; the maximum time which has been observed on the VAX is 1.2 milliseconds. Faster times will occur if either X or Y is small, or if both coordinates are very large.

For the finite-depth case, a distinction is made between the initialization of parameters which depend only on H, using the auxiliary subroutine HGREEN, and the subsequent evaluation in FGREEN. When a new value of H is input to FGREEN, six milliseconds is required by the initial call to HGREEN. This step is avoided for subsequent calls with the value of H unchanged. Thus, in typical usage, where many calls are made successively with different sources and field points but the same value of H, the time required by HGREEN is negligible.

The remaining time for FGREEN depends primarily on the ratio X/H , with a maximum run time of 6 milliseconds occurring when $0.5 < X/H < 0.6$. For smaller values of X/H , the run time will vary between about 2 ms. and 4 ms. as X/H increases from zero to 0.5. For larger values of X/H the run time will decrease in a manner which is inversely proportional to

X/H. Thus for many calls in finite depth the average run times are about 3-4 milliseconds on the VAX.

4. REFERENCES

1. "Algorithms for the oscillatory source potential," by J. N. Newman (submitted to the 16th International Congress of Theoretical and Applied Mechanics).
2. "Double-precision evaluation of the oscillatory source potential," by J. N. Newman, J. Ship Res., in press.

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